Strong topological Rokhlin property of countable groups

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In 1940s, following the previous work of Oxtoby and Ulam, Halmos proved the following result

Theorem (Halmos)

P.m.p. bijections of the standard probability space (i.e. of $([0,1],\lambda)$) that are ergodic and/or weak mixing are 'generic' among all such p.m.p. bijections.

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removing thus fears that such properties might be rare.
Moreover, he also proved

Theorem (Halmos)

A p.m.p. bijection of the standard probability space has a dense conjugacy class (in the space of all p.m.p. bijections) if and only if it is aperiodic.

If there were a single transformation that is generic, then in order to verify which property is generic, it would suffice to check this property for this one single transformation. Unfortunately, in the measurable setting we have:

Theorem (del Junco - 1979)

There is no generic p.m.p. bijection of the standard probability space.

Introduction and history

If there were a single transformation that is generic, then in order to verify which property is generic, it would suffice to check this property for this one single transformation. Unfortunately, in the measurable setting we have:

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The case above corresponds to the action of the integers. We would like to consider actions of more general groups and we also have:

Theorem (Foreman-Weiss - 2004)

For no countable amenable group G, there is a generic action of G on the standard probability space by p.m.p. bijections.

Introduction and history

At least we have:

Theorem (Glasner, Thouvenot, and Weiss - 2006/independently Hjorth - unpublished)

For every countable group G, a generic action of G on the standard probability space by p.m.p. bijections has a dense conjugacy class.

Cantor dynamics

From now on, we consider topological dynamics and more specifically on the Cantor space. Let $\mathrm{Homeo}(2^{\mathbb{N}})$ denote the group of all self-homeomorphisms of the Cantor space equipped with the uniform-convergence topology which makes it a Polish group. If G is a countable group we can identify the set of all continuous actions of G on $2^{\mathbb{N}}$ with the set of homomorphisms from G to $\mathrm{Homeo}(2^{\mathbb{N}})$ which can be further identified with a closed subset of $\mathrm{Homeo}(2^{\mathbb{N}})^G$. Let us denote such a Polish space by $\mathrm{Act}_G(2^{\mathbb{N}})$.

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- $\mathrm{Act}_{\mathbb{Z}}(2^{\mathbb{N}})$ with $\mathrm{Homeo}(2^{\mathbb{N}})$;
- If G is the free group on n generators, then we can identify $\mathrm{Act}_G(2^{\mathbb{N}})$ with $\mathrm{Homeo}(2^{\mathbb{N}})^n$;
- If $G = \mathbb{Z}^d$, for $d \in \mathbb{N}$, we can identify $\mathrm{Act}_{\mathbb{Z}^d}(2^{\mathbb{N}})$ with $\{(g_1, \ldots, g_d) \subseteq \mathrm{Homeo}(2^{\mathbb{N}})^d : g_i g_j = g_j g_i \forall i, j \leq d\}$.

Cantor dynamics

Glasner-Weiss - 2001

There is a Cantor space homeomorphism with a dense conjugacy class. Does there exist a generic Cantor space homeomorphism? That is, does there exist a conjugacy class in $\mathrm{Homeo}(2^{\mathbb{N}})$ that is comeager in the topology of $\mathrm{Homeo}(2^{\mathbb{N}})$?

Glasner-Weiss - 2001

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Apparently, the expectations were not so high and the following came as a surprise:

Theorem (Kechris-Rosendal - 2007)

Yes, there is a generic Cantor space homeomorphism.

Cantor dynamics

The Kechris-Rosendal result sparked such an interest that it has been since then re-proved several times by many different authors. Kechris and Rosendal in their paper considered actions of more general groups on $2^{\mathbb{N}}$ and asked about generic actions. We note that 'generic action' means an action with a comeager conjugacy class.

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Conjugacy classes of group actions

If G is a countable group then the conjugacy class of $\alpha \in \mathrm{Act}_G(2^{\mathbb{N}})$ (let us view $\alpha : G \to \mathrm{Homeo}(2^{\mathbb{N}})$ as a homomorphism) is the set

$$\{\phi\alpha\phi^{-1} \colon \phi \in \mathrm{Homeo}(2^{\mathbb{N}})\},$$

where for $\phi \in \mathrm{Homeo}(2^{\mathbb{N}})$

$$(\phi\alpha\phi^{-1})(g) = \phi\alpha(g)\phi^{-1}.$$

Cantor dynamics

Definition (Hochman - 2012)

Say that a countable group G has *the strong topological Rokhlin property* if $\mathrm{Act}_G(2^{\mathbb{N}})$ contains a generic action.

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$\mathbb{F}_\infty$, the free group on countably-infinitely many generators, does not have the strong topological Rokhlin property.
Let $n \geq 2$. Does $\mathbb{F}_n$ have the strong topological Rokhlin property?

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Theorem (Kwiatkowska - 2012)

For any $n \in \mathbb{N}$, $\mathbb{F}_n$ does have a strong topological Rokhlin property.

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Theorem (Kwiatkowska - 2012)

For any $n \in \mathbb{N}$, $\mathbb{F}_n$ does have a strong topological Rokhlin property.

Theorem (Hochman - 2012)

For any $d \geq 2$, $\mathbb{Z}^d$ does not have a strong topological Rokhlin property.

Cantor dynamics

Theorem (Hochman - 2012)

For any countable group G, $\mathrm{Act}_G(2^{\mathbb{N}})$ contains a dense conjugacy class.
In general, an action with a dense conjugacy class cannot be found constructively. For $\mathbb{Z}$, there is an algorithm constructing the generic action. For $d \geq 2$, there is no algorithm producing an action of $\mathbb{Z}^d$ with a dense conjugacy class - such an action exists because of an undirect argument.

New results

to be mentioned in this talk:

1. Description of groups with the strong topological Rokhlin property in terms of subshifts of finite type/sofic subshifts over such groups.
2. Some examples and non-examples of groups with the strong topological Rokhlin property.
3. Genericity of shadowing (pseudo-orbit tracing property) for countable group Cantor actions.

Strong topological Rokhlin property

Let G be a countable group and let A be a finite set. G acts on the Cantor space A^G (provided that G is infinite) by the 'shift'; i.e. for $g, h \in G$ and $x \in A^G$ we have

$$g \cdot x(h) := x(g^{-1}h).$$

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By $\mathcal{S}_G(A)$ we shall denote the compact space of all closed subshifts of A^G equipped with the Vietoris topology.
Let $X \subseteq A^G$ be a subshift and let $F \subseteq G$ be a finite subset. Set $X_F := \{x \restriction F \colon x \in X\}$. Then

$$N_X^F := \{Y \in \mathcal{S}_G(A) \colon X_F = Y_F\}$$

is a basic open neighborhood of X (defined by F).

Strong topological Rokhlin property

Definition (some important classes of subshifts)

A subshift $X \subseteq A^G$ is *of finite type* if there exists a finite $F \subseteq G$ such that X is the biggest element (with respect to inclusion) of N_X^F.

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A subshift $X \subseteq A^G$ is *of finite type* if there exists a finite $F \subseteq G$ such that X is the biggest element (with respect to inclusion) of N^F_X.
If $X \subseteq A^G$ is any subshift and $F \subseteq G$ a finite set, then there is a biggest element in N^F_X, i.e. a subshift of finite type $Y \subseteq A^G$ such that $X_F = Y_F$.

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Definition

Let G be a group and A be a finite set. Say that a subshift $X \subseteq A^G$ is *projectively isolated* if there exist a finite set B, a subshift $Y \subseteq B^G$, an open neighborhood $\mathcal{U}$ of Y in $\mathcal{S}_G(B)$, and a continuous equivariant map $\phi : Y \to A^G$ such that for every $Y' \in \mathcal{U}$ we have $\phi[Y'] = X$.

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Remark

- If a subshift $X \subseteq A^G$ is isolated in the Vietoris topology, then it is projectively isolated. Take Y to be X, $\mathcal{U}$ to be the isolating neighborhood of X and ϕ to be the identity map.
- If a subshift is isolated, then it must be of finite type.
- If a subshift is projectively isolated, then it must be sofic.

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- If a subshift is isolated, then it must be of finite type.
- If a subshift is projectively isolated, then it must be sofic.

Projectively isolated subshift that is not isolated

Let $x \in 2^{\mathbb{Z}}$ be defined by $x(0) = 0$ and $x(n) = 1$ for $|n| > 0$. Let X be the subshift generated by x. Then X is projectively isolated, but it is not of finite type, so it is not isolated.

Strong topological Rokhlin property

- Every countable group admits an isolated subshift - e.g. $X = \{x\}$, where $x : G \to A$ is constant.

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- If G is finitely generated, then $X = \{x_1, \ldots, x_n\}$, where each x_i is fixed, i.e. constant, is isolated.

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- Every countable group admits an isolated subshift - e.g. $X = \{x\}$, where $x : G \to A$ is constant.
- For some (maybe for all) non-finitely generated groups these are the only projectively isolated subshifts.
- If G is finitely generated, then $X = \{x_1, \ldots, x_n\}$, where each x_i is fixed, i.e. constant, is isolated.
- Non-trivial transitive isolated subshifts exist on free groups and their products with any finitely generated groups (probably free groups can be generalized in this statement to shortlex automatic groups, in particular hyperbolic groups).

Strong topological Rokhlin property

Theorem

A countable group G has the strong topological Rokhlin property if and only if for every finite set A the set of projectively isolated subshifts is dense in $\mathcal{S}_G(A)$.

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Remark

If a countable group G does have the strong topological Rokhlin property, then the generic action is constructed as a 'generic' inverse limit of projectively isolated subshifts over G.

It allows us to provide some new examples and non-examples and new proofs of some previous results.

Strong topological Rokhlin property

Examples

$\bigstar_{i \leq n} G_i$, where for $i \leq n$, G_i is either finite or cyclic.

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Selection of non-examples

- Any finitely generated nilpotent group that is not (virtually) $\mathbb{Z}$.
- $G_1 \times G_2 \times G_3$, where for $i \in \{1, 2, 3\}$, G_i is recursively presented, and G_1 is indicable.
- 'Many' non-finitely generated groups.

Theorem

Let G be any countable amenable group. Then the set of those actions of G on $2^{\mathbb{N}}$ whose topological entropy is zero is dense G_δ in $\mathrm{Act}_G(2^{\mathbb{N}})$.

Dynamical properties of generic actions

Theorem

Let G be any countable amenable group. Then the set of those actions of G on $2^{\mathbb{N}}$ whose topological entropy is zero is dense G_δ in $\mathrm{Act}_G(2^{\mathbb{N}})$.

This can be proved using the result of Frisch and Tamuz that zero entropy is comeager in $\mathcal{S}_G(A)$ for every amenable G and A.
I am not aware of any infinite amenable group except $\mathbb{Z}$ that would admit a generic action on $2^{\mathbb{N}}$.

Definition

Let G be a countable group and let $\alpha \in \mathrm{Act}_G(2^{\mathbb{N}})$. We say that α has *shadowing* (or *the pseudo-orbit tracing property*) if there exists a finite subset $S \subseteq G$ such that for every clopen partition $\mathcal{P}$ of $2^{\mathbb{N}}$ there exists a refinement $\mathcal{P}' \preceq \mathcal{P}$ so that for every $(x_g)_{g \in G} \subseteq 2^{\mathbb{N}}$ if for every $g \in G$ and $s \in S$

$$x_{sg} \text{ and } \alpha(s)x_g \text{ lie in the same element of } \mathcal{P}'$$

then there is $x \in 2^{\mathbb{N}}$ such that for every $g \in G$

$$\alpha(g)x \text{ and } x_g \text{ lie in the same element of } \mathcal{P}.$$

If G is finitely generated, then the property above holds for any finite generating set.

Genericity and shadowing

Theorem (Walters - $\mathbb{Z}$ (1982), Oprocha - $\mathbb{Z}^d$ (2008), Chung& Lee - general case (2018))

Let G be a countable group. A subshift $X \subseteq A^G$ has shadowing if and only if it is of finite type.

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There has been an extensive research on genericity of dynamical systems with shadowing. We mention

Theorem (Bernardes, Darji (2012))

Shadowing is generic for homeomorphisms of the Cantor space.

Theorem

Let G be a finitely generated group. Then shadowing is generic in $\mathrm{Act}_G(2^{\mathbb{N}})$ if and only if G has the strong topological Rokhlin property.

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Corollary

Shadowing is generic in $\mathrm{Act}_G(2^{\mathbb{N}})$, for $G = \bigstar_{i\leq n} G_i$, where for $i \leq n$, G_i is either finite or cyclic

Genericity and shadowing

While it is relatively straightforward to show that shadowing is preserved by taking inverse limits, the generic action is by construction, in general, an inverse limit of sofic subshifts, which do not have shadowing. Although in some special cases, such as $\mathbb{Z}$ or $\mathbb{F}_n$, it is an inverse limit of subshifts of finite type.

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While it is relatively straightforward to show that shadowing is preserved by taking inverse limits, the generic action is by construction, in general, an inverse limit of sofic subshifts, which do not have shadowing. Although in some special cases, such as $\mathbb{Z}$ or $\mathbb{F}_n$, it is an inverse limit of subshifts of finite type.

Definition (Good,Meddaugh (2020)/Lin, Chen, Zhou (2022))

Let G be a countable group and let $\left((X_n), \phi_n^m : X_n \to X_m\right)_{n \geq m \in \mathbb{N}}$ be an inverse system of compact G-spaces where the bonding maps are not necessarily onto. Say that the system has the *Mittag-Leffler property* if for every $m \in \mathbb{N}$ there is $n_0 \in \mathbb{N}$ such that for all $n, n' \geq n_0$

$$\phi_n^m[X_n] = \phi_{n'}^m[X_{n'}].$$

Genericity and shadowing

Theorem (Good,Meddaugh (2020)/Lin, Chen, Zhou (2022))

Let G be a finitely generated group and $\alpha \in \mathrm{Act}_G(2^{\mathbb{N}})$. Then α has shadowing if and only if it is an inverse limit of a system of subshifts of finite type satisfying the Mittag-Leffler property.

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Idea of the proof of shadowing for generic actions

The generic action is an inverse limit of a system of subshifts of finite type satisfying the Mittag-Leffler property.

Genericity and shadowing

Theorem

Let G be a countable group that does not have the strong topological Rokhlin property (e.g. $\mathbb{Z}^d$, for $d \geq 2$, nilpotent groups, etc.). Then shadowing is not generic in $\mathrm{Act}_G(2^{\mathbb{N}})$; in fact, the set of actions $\alpha \in \mathrm{Act}_G(2^{\mathbb{N}})$ that have shadowing is a dense meager set in $\mathrm{Act}_G(2^{\mathbb{N}})$.

Genericity and shadowing

Idea of the proof. Let G be such that it does not have the strong topological Rokhlin property.

1. There exist a finite set A and an non-empty open set $\mathcal{U} \subseteq \mathcal{S}_G(A)$ such that for every $X \in \mathcal{U}$ the set of those $\alpha \in \mathrm{Act}_G(2^{\mathbb{N}})$ that don't factor onto X is comeager.

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2. The set of those $\alpha \in \mathrm{Act}_G(2^{\mathbb{N}})$ that have a factor in $\mathcal{U}$ is non-empty open.

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4. There are countably many sofic subshifts inside $\mathcal{U}$.

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4. There are countably many sofic subshifts inside $\mathcal{U}$.

If shadowing were generic in $\mathrm{Act}_G(2^{\mathbb{N}})$, by (2) non-meager many actions with shadowing would factor inside $\mathcal{U}$. By (3), each of these non-meager many actions would factor onto a sofic subshift. By (1) and (4), the set of actions that don't factor onto any sofic subshift in $\mathcal{U}$ is comeager. Contradiction. The meagerness follows from a 0-1 law.

Locally compact group actions

For any locally compact Polish group G there is a Polish space of continuous actions of G on $2^{\mathbb{N}}$. Only totally disconnected groups can act faithfully on $2^{\mathbb{N}}$ and we can characterize pro-countable locally compact Polish groups that have the strong topological Rokhlin property. In particular, we can prove:

Theorem

Every pro-finite metrizable group (i.e. every group homeomorphic to $2^{\mathbb{N}}$) has the strong topological Rokhlin property.

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THANK YOU FOR YOUR ATTENTION!

Reference: M. Doucha, *Strong topological Rokhlin property, shadowing, and symbolic dynamics of countable groups*, arXiv: arXiv:2211.08145 [math.DS]